BASICS OF GEOMETRY

CIRCLES THEOREMS

Geometry is a trending topic in Bank exams. In the series of sharing Geometry basics, today I am sharing rules related to Circles :-



CHORDS OF A CIRCLE

The longest chord in a circle is diameter.



AB, CD, EF are the different chords of the circle. You can see that as you move away from the centre the length of the chord gets smaller and smaller but as you move towards the centre, the length of the chord becomes greater and greater.

Based on the same logic , there can be many questions like :



2. Both the chords are on the same side of the center.

Let us solve it with first possibility



Here one theorem comes into existence that says that any perpendicular drawn from the center of the circle to chord, divides the chord.

Since OP is a perpendicular on the chord AB Therefore AP=PB =12 In triangle OPB by Pythagorean Theorem $OP^2 = OB^2 - PB^2$ $= 13^2 - 12^2$ = 169 - 144 = 25 OP= 5 cm Similarly since OR is perpendicular to the chord AB Therefore CR=RD =5 cm In triangle ORD by Pythagorean theorem $OR^2 = OD^2 - RD^2$ = 13²-5² = 169-25 =144 OR = 12

Therefore the distance between the chords AB and CD = OP+OR =5+12=17 cm Now let us do it with second possibility



ANGLES IN A CIRCLE

A single chord divides the circle into two segments.



ANGLES IN THE MAJOR SEGEMENT

RULE 1:

According rule the angles made in the same segments are equal.

OR

Angles made by the same chord in the same segment are always equal.



Angles made by the chord at center are twice the angle made at the circumference.



ANGLES IN THE MINOR SEGMENT



Please note that in case of minor segment, the angle that is twic the angle at circumference.It is actually a reflex angel because it is greater than. 180°

TRIANGLES THEOREMS

THEOREM 3: ANGLE SUM PROPERTY OF A TRIANGLE.

The sum of the angles of a triangle is 180° .



In the picture above, PQR is a triangle with angles 1, 2 and 3 Then according to the theorem Angle 1+Angle 2 +Angle 3 =180⁰

THEOREM 4

If a side of a triangle is produced then the exterior angle so formed is equal to the sum of two interior opposite angles.



In the picture above XYZ is a triangle whose side YZ is extended to R. 1, 2, and 3 are the interior angles of a triangle. Angle 1 and Angle 2 are the interior angles opposite to the exterior angle 4 Then according to the theorem Angle 4 = Angle 1+ Angle 2

Let us do some questions based on these theorems.

Ques. - In the figure if QT is perpendicular PR , Angle TQR = 40° and Angle SPR = 30° , find x and y .



Solution : In triangle QTR Angle TQR +Angle QRT +Angle QTR = 180° $40^{\circ} + y + 90^{\circ} = 180^{\circ}$ $y = 180^{\circ} - 130^{\circ}$ $= 50^{\circ}$ Angle QSP = Angle SPR +Angle SRP

<u>Reason:Exterior angle</u> = sum of interior opposite angles

 $x = 30^{\circ} + y$ $x = 30^{\circ} + 50^{\circ}$ $x = 80^{\circ}$

Ques. - In the figure below XY II MN , Angle YXZ =350 and angle ZMN =530, find angle MZN.



Solution:

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We know that XY is parallel to MN.

Angle MNZ = Angle ZXY (alternate interior angles)

= 35^{\circ}

Now in triangle MZN

Angle ZMN +Angle MNZ +Angle MZN =180^{\circ}

53^{\circ} + 35^{\circ} +Angle MZN =180^{\circ}

Angle MZN = 180^{\circ} - 88^{\circ}

Angle =92^{\circ}
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Ques - In the figure given below If PQ and RS intersect at T, such that angle PRT = 40° , angle RPT= 95° and angle TSQ= 75° , find SQT.



Ques. In the figure if PQ is perpendicular to PS, PQ II SR Angle SQR = 28° and Angle QRT = 65° , then find the values of x and y.



Solution:

Since PQ II SR Angle QRT = Angle PQR (alternate interior angles) $65^{\circ} = x + 28^{\circ}$ $X = 65^{\circ} - 28^{\circ}$ $= 37^{\circ}$ In triangle PQS Angle PSQ +Angle PQS + QPS = 180° (angle sum property) $Y + x + 90^{\circ} = 180^{\circ}$ $Y + 37^{\circ} + 90^{\circ} = 180^{\circ}$ $Y = 180^{\circ} - 127^{\circ}$ $= 53^{\circ}$



COMPLEMENTARY ANGLES:

Two angles whose sum is 90 degree are called Complementary Angles.

For example 40 degree and 50 degree

Their sum =40 degree +50 degree = 90 degree

SUPPLEMENTARY ANGLES:

Two angles whose sum is 180 degree are called Supplementary Angles.

For example 100 degree and 80 degree

Their sum =100 degree + 80 degree = 180 degree

LINEAR PAIRS:

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Two angles on a given line are called linear if their sum is 180 degree. Look at the picture below.



In the above figure angle ABD and angle CBD are linear pair because their sum is 180 degree.

VERTICALLY OPPOSITE ANGLES:

Vertically opposite angles are formed when two lines, say AB and CD intersect each other at a point O. There are two pairs of vertically opposite angles and they are always equal.



One pair is angle AOD and angle COB Angle AOD = Angle COB (Vertically opposite angles) The other pair is angle AOC and angle BOD

Angle AOC = Angle BOD (Vertically opposite angles)

Note : If two lines intersect each vertically opposite angles are equal.

TRANSVERSAL

A line which intersects two or more lines at different points is called a transversal. Let us understand this:



In the above picture line I intersects lines m and n at points P and Q respectively. Thus line I is a transversal for lines m and n. By observing the picture, we can see that there are four angles formed at each point P and Q.

Angle 1 , Angle 2 , Angle 7, Angle 8 are called exterior angles, while Angle 3, Angle 4, Angle 5 and Angle 6 are called interior angles.



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INTRODUCTION:

We usually make faces when asked to go through the topic "MENSURATION", the sole reason behind this is "THE FORMULAS" but wait what if we had to just understand everything and apply it in practical life ?? Read it right, today we are going to get our basic concepts cleared under this topic with few examples supporting the concepts.

Before heading to the formulas, let's know the meaning of the word MENSURATION. The word mensuration means measurement, this is a branch of mathematics which helps us in dealing with the study of plane and solid figures, their area, volume, and related parameters.

IMPORTANT DEFINITIONS

The extent or measurement of a surface or piece of land or the total amount of space inside the boundary of flat objects is Area. The shaded region in the following image is the area of the rectangle.

1. AREA



2. PERIMETER

The perimeter of any 2-dimensional object is the measure of the covering the inner area.

Here the dotted lines represent the boundary

lf

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\rightarrow I =10cm and b=4 then perimeter = 2 (I+b)
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→ 2(10+4)

\rightarrow 2 x14

 \rightarrow 28 cm (add the boundary measurements you get 28)



3. CIRCUMFERENCE

When you have any curved geometric figure then their distance around the body will be called as circumference, especially for the circle.

4. DIMENSIONAL OBJECTS

A three-dimensional shape is a solid shape that has height and depth. For example, a sphere and a cube are three-dimensional, but a circle and a square are not.

5. CURVED SURFACE AREA:

The surfaces which are not flat, are called curved surface.

The lateral surface is the area of the vertical faces of the solid.

Curved surfaces do not include the top or bottom are, for example, CSA of the cylinder will not include the area of upper and lower circle.

Whereas the total surface area (TSA) will include both the area of both upper and bottom portion.

LET'S START WITH THE FORMULAS:

- Area of a square = side x side = (side)2
- **Perimeter of square** will be total of its four sides measurement = 4(side)
- <u>Area of a rectangle</u> = I x b (similar to square where we multiple sides of the same measurement but rectangle has two different measurements)
- The perimeter of a rectangle = 2 (I+b) or simply add the sides.
- **Perimeter of a circle** = 2⊓r

Area of a circle = $\Box r^2$ (where r is the radius)

NOW, WHAT IS **n**?

□ is the relationship between distance around the circle i.e. (circumference) and distance across the circle whose value is 22/7 or 3.14.....

For any circle, the ratio remains same no matter how big or small it is (radius will not matter) Example:

D stands for diameter (2 x radius = diameter)

D=7

D=14

Circumference=22



D=21 circumference=66



So comparing the relationship between circumference of all these circles we get same value

 \rightarrow C1/D1= 22/7

\rightarrow C2/D2=44/14=22/7

Circumference may increase but the ratio remains same.

In simple terms, if you wrap the diameter of any circle to its outer boundary you will need approximately 3.14..... times diameter.

FORMULAE FOR DIFFERENT SHAPES

CYLINDERS



Volume of Cylinder = ⊓r2h

Suppose if someone asks you to find the area of a coin you can simply use the formula of circle neglecting the height of the coin. Same way if someone asks you to keep one coin over the another and ask you to calculate the area then you will have to consider the number of coins or say the height of the cylinder you will form out of the coins.

The base circle area is ⊓r2 multiplied with h will give you total volume.

<u>Curved Surface Area of a cylinder</u> = perimeter of base x height = $2 \Box rh$

(take the circumference of the bottom circle multiply it h, will give you the total curved surface area) <u>Total Surface Area</u> = Curved Surface Area + 2 base x height

Total surface area will be total of curved surface area and the base and the upper circle area which are of the same area so take either of the two base or upper)

CONE



It takes the volume of three cones to equal one cylinder, all having the same height and radii of a cylinder.

So when **volume of cylinder** = $\sqcap r2h$

Then the **volume of cone** = $1/3 \times \pi r^2h$

Curved Surface Area of cone: ⊓rl

- \rightarrow R=radius of the cone
- \rightarrow H = height
- \rightarrow L=slanting height which you can calculate using
- \rightarrow l2 = r2 + h2

In cone, the circle circumference is 2 Ir and r is the radius

when a cone is opened, you get a shape of an arc making an imaginary circle with L as radius because L is the slant height and edge of the cone and for the imaginary circle the circumference is 2Π and area of the circle will be Π ²

Comparing their ratios will give you the area:



SPHERE:



A sphere is a perfectly round geometrical object in

three- dimensional space, best example cricket ball or football. spheres do not have any bottom or top is totally curvy. So both lateral/curved surface and total surface are equal

Total Surface Area = Curved Surface Area

It becomes difficult to find area of curvy objects so with help of cylinder we can understand this:

(proved by greek mathematician Archimedes)

You can fit a sphere in cylinder (with same radii and height)

Then the lateral surface of cylinder = area of sphere

Curved Surface Area of Cylinder = 2⊓rh

Curved Surface Area of sphere = 2⊓rh

(r is same for both, we do not have h in sphere so we consider diameter to be h =2r)

<u>Curved Surface Area</u> = 2⊓r(2r) substituting h=2r

Curved Surface Area= 4⊓r2

Volume of sphere =4/3 r3 (sphere is 1/3rd of cursed surface of sphere into r)

Volume of hemisphere = $2/3 2 \square r3$ (half of the volume of sphere)

<u>Curved Surface Area of hemisphere</u> = $4 \square r^2 / 2$ (half of spheres) = $2 \square r^2$



NOTE: Remember friends TSA of a hemisphere is not same as CSA (don't do this mistake of halving the TSA of the sphere because hemisphere has one side circle so you will have to add the area of the circle too:

 \rightarrow Total Surface Area = $2 \Box r^2 + \Box r^2 = 3 \Box r^2$



QUESTION1

A road roller of diameter 1.75m and length 1m has to press a ground of area 1100 SQ. meter .how many revolutions does it make?

SOLUTION:

We know a roller is in the shape of a cylinder, now when you roll a cylinder one revolution it makes will be equal to the curved surface area of the cylinder (you cannot take volume here because it not about how much the cylinder holds in it nor it is about the total surface of cylinder) r=1.75/2 h=1Area covered in one revolution= curved surface area = $2 \pi h$ Total area to be pressed =1100 SQ meter

Number of revolutions= Total area to be pressed/curved surface area

= 1100 / $2 \Box rh$ (substituting value)

Ans: -Number of revolutions = 200

QUESTION2

A rectangular sheet of paper of length 10 cm and breadth 24cm is rolled end to end to form a right circular cylinder of height 8 cm. find the volume of the cylinder.

SOLUTION:

When you roll a rectangle sheet to a cylinder shape, the base forms a circle. We need to find out what the circumference of this circle is to get the radius, which will later be utilised in volume.

Circumference = $2 \Box r$ (here clearly they have given 10 must be the height so 24 is the circumference)

 \rightarrow 24 = 2*22/7*r →24*7/22*1/2=r →r = 42/11cm

Ans:-volume = ⊓r2h = 458.18 cu.cm

QUESTION3

A right cylindrical vessel of 15 cm radius is filled with water. Solid spheres of diameter 6 cm are dropped one by one into it. The spheres are dropped until the water level in the vessel rises by 8 cm. then, how many solid spheres are dropped into the vessel?

SOLUTION:

Let n balls be dropped into the cylinder Volume of n balls will be the total increase in water level in the vessel Given d=6 ;so r = 3 ; rise in water level h=8 $\rightarrow N^{4}/3^{\pi}\Pi^{3} = \Pi r^{2}h$ $\rightarrow N^{4}/3 \Pi (3)^{3} = \Pi * 152 * 8$ Ans:- $\rightarrow N = 50$ balls

QUESTION4

If a cone and a sphere have equal radii and have equal volumes, then what is the ratio between the height of the cone and diameter of the sphere?

SOLUTION:

Let h be the height of the cone and r be the radius of the sphere as well as the radius of the base of the cone, clearly given that volume of the sphere = volume of the cone

→i.e. 4/3 ⊓r3 = 1/3 ⊓r2 h

 \rightarrow 2r=h/2=d

Therefore the height of the cone diameter of the sphere

4r:2r

Ans:-required ratio 4:2= 2:1

QUESTION5

The length of a rectangular plot is 60% more than its breadth. if the difference between the length and the breadth of that rectangle is 24cm. what is the area of that rectangle?

SOLUTION:

Let breadth = x cm then length= 160*x/100 = 8/5*x \rightarrow So 8/5*x - x = 24 \rightarrow X=40 Length =64 breadth =40 **Ans:-**Then area will be =64*40= 2560

QUESTION6

A spherical ball of 6cm diameter is melted into a cone with base 12cm in diameter. find its height.

SOLUTION:

Here diameter is given so radius will be d/2

→4/3⊓r3 = 1/3⊓R2h

→4*33 = 62 * h

Ans:-→H= 3cm

QUESTION7

4 containers are in the shape of a sphere of radius 7cm. find the cost of panting at Rs.2 per square metre and filling them with a liquid costing Rs.9per cubic cm.

SOLUTION:

Surface area of the sphere = $4 \square r^2$ Surface area of the containers = 4*22/7*72 \rightarrow = 616 cm² Cost of painting =Rs.2 * 616(surface area)= Rs.1232

For filling the containers you need to calculate the volume of the sphere using the formula $4/3 \square r^3$ Ans:-Cost of filling = Rs.9* volume of sphere = Rs.9* $4/3^{*} \square ^{*}73$ = Rs.12936

BASICS OF GEOMETRY

Geometry is a very important topic the aspirants who are preparing for the SSC, IBPS, SBI exams. One can expect good number of questions from this topic. Now let us discuss the basic concepts which an aspirant should be clear before solving the concepts. Please make sure that all the basic concepts are clear before solving the problems otherwise it will be difficult to solve an easy problem.

THE MORE YOU IN GEOMETRY THE MORE YOU SCORE.

Let us start with the basic unit of Geometry i.e Point.

POINT

A point is the basic unit of the geometry which is visually represented by a DOT (.). Also denoted by Capital letters for example.

Ex:

COLLINEAR POINTS: Two Or more points lie on the same line and in the same plane are known as collinear points.



Points A,b,c,d are lying on the straight line in the same plane and thus known as collinear points. **NON-COLLINEAR POINTS:**Points which do not lie on the same line are known are Non-collinear points.

А



Points x,y,z,a are not lying on the same line but are on the same plane , thus they are known as noncollinear points.

PLANE: A plane is a two-dimensional figure that extends infinitely.



look like.



AB- A ray is always represented in this way- First letter is the ending point (A).

CONCURRENT LINES: Three or more lines pass through same point are called concurrent lines and the common point is known as point of concurrence.



In the above figure a,b,c,d,e are the lines intersecting at Point 'O'. Point 'O' is known as point of concurrence and all these lines are known as concurrent lines.

PARALLEL LINES: Two lines in the same plane are said to be parallel if they never meet even though they are extended in either direction. They remain same distant for whole length. It is denoted by

When a line cuts the two parallel lines then it is known as transversal.



AB and **YZ** are the two parallel lines and **PQ** is the transversal which intersects both AB and YZ. Transversal makes **4 pairs of -CORRESPONDIGN ANGLES**, Which are equal in value. They are

<u>_1=_5,_2=_6,_3=_7,_4=_8</u>

Transversal makes 4 pairs of ALTERNATE ANGLES which are equal

$\angle 1 = \angle 8$, $\angle 2 = \angle 7$, $\angle 3 = \angle 6$, $\angle 4 = \angle 5$

*Angles 3,4,5,6 are known as the interior angles. Sum of the interior angles on the same side of the transversal is equal to two right angles i.e 180°.

$\angle 4 + \angle 6 = 180, \ \angle 3 + \angle 5 = 180$

*Angles 1,2,7,8 are known as the exterior angles. Sum of the exterior angles on the same side also equal to two right angles i.e 180°.

2+**2**8=180, **2**1+**2**7=180

PERPENDICULAR LINES: A line meeting the another line at right angle or at 90°.



When two rays are joined with a common end point, then an angle is formed. The common end point is called as the **vertex** of the angle. The rays are known as the **sides** of the angle.



TYPES OF ANGLES: ANGLES BASED ON THE VALUE OF THE ANGLE.

Acute Angle: An angle whose value lies between 0° and 90° is called as an acute angle. Right Angle: An angle whose value is 90° is called right angle.

Obtuse Angle: An angle whose value lies between 90° and 180° is known as obtuse angle.

Straight Angle: An angle whose value is 180° is known as straight angle

Reflex angle: An angle whose value lies between **180**° and **360**° is known as reflex angle.

In the above figure angle $XON = b^{\circ}$ is the reflex angle.

Complete angle: An angle whose value is **360**° is known as complete angle.

Complimentary angle: If the sum of Two angles is **90**° then they are known as complimentary angle.

Ex: If angle A = 50° is compliment to angle B then the value of B=(90°-50°)=40°

Supplementary angle: If the sum of the angles is **180°** then they are known as supplementary angle.

Adjacent Angles: Angles having one side as common is known as adjacent angles.



Here angles **a**,**b** are having a common side and thus these are known as adjacent angles.

Linear Pair angles: Two angles form a linear if they are having common vertex and sum of the angles is 180°.

Note: All adjacent angles are not linear pair angles.

Ex: In the given figure angle XOZ and YOZ form a linear pair if a-b=80° find the value of a and b.



Given a-b=80°

a+b=180° as it is clearly stated that a and b are linear pair.

BY solving both the equations we get ,

2a=260°,

 $a=130^{\circ}$ and the value of $b=50^{\circ}$

VERTICALLY OPPOSITE ANGLES: When two lines intersect each other four angles are formed . The angles lie on the opposite side of the intersecting point are called as vertically opposite angles.



Here angles **p** and **q** are opposite to each other similarly **s** and **r** are opposite to each other. Vertically opposite angles are always equal i.e **p**=**q**

r=s..

Angle Bisector: A line which cuts angle into two equal angles is called an angle bisector. An angle bisector can be of two types

1.Internal angle bisector

2. External angle bisector.

Internal angle Bisector:



From the above figure it is clear that what is meant by a angle bisector. **or is an angle bisector** which divided the angle **qos** into two equal angles.

TRIANGLE: A PLANE FIGURE BOUNDED BY THREE STRAIGHT LINES IS CALLED A TRIANGLE.

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The above figure shows how a triangle looks like.

Types Of Triangles: Based on Angles

Acute angled Triangle: A triangle which each angle is less than 90° is called as acute angled triangle. Right angled Traingle: A triangle in which one of the three angles Is 90° is known as right angled triangle.

Obtuse angled Triangle: A triangle in which one of the angle lies between 90° and 180° is known as obtuse angled triangle.

Types of Triangle: Based on Sides

Equilateral Triangle: A triangle in which all the three sides are equal is known as equilateral triangle. **Scalene Triangle:** A triangle in which all the three sides are of different length is known as scalene triangle.

Isosceles Triangle: A triangle in which two out of three sides are equal is known as Isosceles triangle. **Congruency of Triangle:**

In Geometry two figures are congruent or similar if they have same size and shape.

Two line segments are said to be congruent if they have same length

Two angles are congruent if they have same measure

Two circles are congruent if they have same diameter

In the same way two triangles can be congruent in the following cases

SSS: In this case if three sides of one triangle are equal to the three sides of another triangle then they are said to be congruent .

SAS: In this case if two sides and an angle made by one triangle is equal to the two sides and angle made by another triangle then they are said to be congruent.

ASA: Two angles and a side of triangle is equal to two angles and a side of another triangle then the triangles are said to be congruent.

If the corresponding angles of two triangles are equal and differ in the length of the sides. Then



Let us discuss more about properties of triangle, orthocenter, incenter , polygons in the next article....ss